

Title: QUANTIZATION OF CONSTANTLY CURVED TETRAHEDRON

Abstract:

Quantum tetrahedron is a key building block in the theory of Loop Quantum Gravity (LQG) and plays a crucial role in the boundary states of the spinfoam amplitude of LQG. In LQG with vanishing cosmological constant, the physical Hilbert space of the quantum flat tetrahedron is the 4-valent $SU(2)$ intertwiner space labeled by irreducible representation, each assigned to a face of the quantum flat tetrahedron. Furthermore, the space is the solution space of the quantum flat closure condition. The area spectrum of each face of the quantum flat tetrahedron is discrete and is characterized by a spin label. Classically, the correspondence between a set of solutions of flat closure condition and flat tetrahedron is guaranteed by the Minkowski theorem. This theorem has been generalized to the curved case, where a curved closure condition applies. The curved Minkowski theorem allows us to reconstruct homogeneously curved tetrahedra (spherical or hyperbolic tetrahedra) from a family of four $SU(2)$ holonomies that satisfy the curved closure condition. Although the quantization of the closure condition for a flat tetrahedron has been extensively studied in LQG, the quantization of the curved closure condition and curved tetrahedron has not been explored yet. The homogeneously curved tetrahedron has played an important role in the recent construction of the spinfoam model with cosmological constant in 3+1 dimensional LQG. It is anticipated that the quantization of a curved tetrahedron should define the building block for the boundary Hilbert space of the spinfoam model. We study the quantization of the curved closure condition and a homogeneously curved tetrahedron. The solution space of the curved closure condition coincides with the moduli space of $SU(2)$ flat connections on a four-puncture sphere. By using the combinatorial quantization, we prove that the solution space of the quantum curved closure condition is the intertwiners space of $U_q(\mathfrak{su}(2))$ and quantum area operators are well defined. A set of coherent intertwiners are constructed directly in the intertwiners space and they possess a semi-classical interpretation. The coherent intertwiner we constructed is the analog of Livine-Speziale coherent intertwiner of the flat tetrahedron.