ABSTRACT

On the paraxial approximation in quantum optics

We examine how best to associate quantum states of a single particle to modes of a narrowly collimated beam of classical radiation modeled in the paraxial approximation, both for scalar particles and for photons. Our analysis stresses the importance of the relationship between the inner product used to define orthogonal modes of the paraxial beam, on the one hand, and the inner product underlying the statistical interpretation of the quantum theory, on the other. While several candidates for such an association have been proposed in the literature, we argue that one of them is uniquely well suited to the task. Specifically, the mapping from beam modes to "henochromatic" fields on spacetime is unique within a large class of similar mappings in that it is unitary in a mathematically precise sense. We also show that the single-particle quantum states associated to the orthogonal modes of a classical beam in the henochromatic approach are not only orthogonal, but also complete in the quantum Hilbert space.